Dual-Control Guidance Strategy for Homing Interceptors Taking Angle-Only Measurements

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A discrete dual-control homing guidance law is proposed to improve intercept capabilities when angle-only measurements are available. A two-maneuver intercept is considered where the first maneuver inserts a measurable line-of-sight (LOS) rate to enhance the estimate of a key guidance parameter prior to the terminal maneuver used to null intercept miss. A numerical technique is proposed to evaluate the gradient of the variance of the chosen parameter with respect to a velocity correction. A first-order search is used to find a velocity correction that minimizes or substantially reduces the terminal variance. The algorithm is extended to multiple-burn scenarios and is evaluated using the terminal estimation error in miss, velocity correction, and time-to-go, respectively, as the control criterion. The results yield interesting insights into the information content in the angle measurements. More importantly, they indicate that the dual-control guidance law suggested here could reduce rms angle-measurement accuracy requirements by a factor of 10 or more, compared to that needed by a more traditional guidance approach, while still maintining terminal intercept performance.

Introduction

TRADITIONALLY, homing intercept problems have been solved through the use of deterministic guidance strategies. Proportional navigation laws, using interceptor acceleration to null the measured angular line-of-sight (LOS) rate, exhibit superior performance and implementation simplicity at short range. These missions often allow for measurement of range and/or range rate. It has become apparent that in cases where range information is unavailable, long ranges, for example, the performance of interceptors using proportional navigation can be degraded substantially. This is especially noticeable in cases where angle measurements are poor or unavailable during portions of the engagement and when the measurable LOS rate is small. It should be clear that minimizing intercept miss involves obtaining good information from the available measurements to guide the interceptor to its target. This fact motivates the use of intercept guidance strategies that make maneuvers intended to enhance the measurement information content early in the engagement in order to improve guidance-parameter estimates to make the final maneuvers.

The fact that the control can have a dual effect in nonlinear stochastic systems, i.e., that the control can affect the uncertainty as well as the state, is well known.[1-7] In general, the optimal solution to nonlinear stochastic control problems invokes the use of dynamic programming. Here, the optimal cost-to-go is defined by

$$I_t = \min_u J_t[Y(t),\ U(t), u]$$

where J_t is the expected cost for a future control policy u, conditioned over the information contained in the past measurements $Y(t)$ and over the past control history $U(t)$. In general, then, the optimal control is a function of the past measurements and controls. However, implicit in the optimization process is the fact that measurements will be taken in the future, and that the present and future controls may affect the information content of these measurements. In the general nonlinear stochastic control problem, the interaction between the estimation and control processes is such that the optimal control is a dual control that enhances the information content in the future observation program in order to improve regulatory performance.

Naturally, much of the theoretical work done thus far has focused on suboptimal solutions to the dynamic programming problem, because of the inordinate amount of computer storage and computation required to obtain the optimal solution. In Refs. 1-4, for example, Tse and Bar-Shalom show how the linearization of the stochastic cost functional can be performed for a scalar control to preserve the coupling between the equations governing the state and covariance propagation. They show how a line search can be used to find a near-optimum "wide-sense" control law. This formulation is time-consuming, however, since the covariance and the adjoint equation, respectively, must be propagated forward and backward in time in order to evaluate the expected cost for each point in the line search. To summarize, the work that has been done to date generally linearizes the problem formulation appropriately so that linear control theory can be applied to the solution.

This paper discusses an alternate method of improving homing interception performance when the measurement (angle-only) uncertainty is the predominant source of noise in the system† and when only discrete impulse corrections are considered. The technique assumes that two maneuvers are made. The first is used to enhance the expected quality of an important user-selected guidance-parameter estimate prior to the terminal maneuver. The terminal maneuver nulls the estimated miss, using a traditional homing guidance correction.

The basic concept is introduced in Ref. 2 for the continuous homing problem in two dimensions. In Ref. 2, the future maneuvers are determined by an optimal feedback law developed along the nominal trajectory for a given quadratic cost function of the state and controls. A guidance law is proposed in this paper which makes improvements in the final estimate of a given guidance parameter that is key to the performance of the terminal maneuver. The optimization procedure is simple and more direct because it involves minimization of parameters that are more familiar to the guidance-law designer than quadratic penalty functions, which require "artificial" weighting of the control and state variables.

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†Random target maneuvers are not considered.

The practical motivation behind this strategy is simple. It is well known that proportional navigation or other similar guidance laws perform well when the trajectory-error volume and, particularly, the estimation error of the time-to-intercept are small. In fact, it has been shown that the continuous proportional navigation law is an optimal "minimum-effort" control.[8] In scenarios where the initial error volume and, specifically, the range/range-rate estimation errors are large, the practical guidance objective is to reduce the estimation errors early enough so that the performance of the terminal maneuver and fusing mechanism is not degraded.

This guidance configuration naturally has the attributes of the dual-control solution. But it avoids computational complexity, since the form of the assumed terminal burn is known. It is not necessary to compute the sweep variables needed to assess the impact of future controls. The velocity correction computation for the first maneuver is predictive; only the gradient of the user-supplied scalar information function (with respect to a velocity correction) needs to be calculated. Unless the estimated fuel consumption exceeds the allowable limit, the guidance correction is independent of the terminal maneuver. The technique is predictive in nature, because the gradient calculation is conditioned over the assumed future-measurement history, which is a function of the predicted trajectory after the impulse velocity correction. Future sections will define the three-dimensional homing problem, discuss a simple and practical implementation of the "two-maneuver" dual-control concept, and verify its performance on a simulated homing intercept.

Homing Guidance Problem Formulation for the Case of Discrete Corrections

Figure 1 illustrates the typical interceptor-target geometry. Here, the target is considered to be unpowered and is subject to the same body forces as the interceptor. The target is taken to be the origin of a relative coordinate system. The relative acceleration of the interceptor and target is due only to the interceptor acceleration. For the case of impulse corrections, the control objective is to apply velocity corrections at discrete times during the engagement to null the miss. Since the miss can be estimated only by processing the angle data, several corrections may be needed to zero the miss effectively.

The estimation/control problem can be formulated in either polar or Cartesian coordinates. A discussion of the advantages and disadvantages of each coordinate system follows.

Polar-Coordinate Formulation

In the polar-coordinate formulation, the system states include the range $|r_r|$, azimuth angle θ, elevation angle ϕ, and their corresponding rates. The state dynamics in this formulation are nonlinear, whereas the two angle measurements are linear and corrupted by noise. The equations for an N-maneuver engagement are

$$\dot{x} = f(x) + \sum_{i=1}^{N} u(x,a)\,\delta(t-t_i)$$

where $x^T = [\,|r|\,\theta\,\phi\,|\dot{r}|\,\dot{\theta}\,\dot{\phi}\,]$

$$z = \begin{bmatrix} \theta \\ \phi \end{bmatrix} + \nu(t)$$

for the continuous measurement case, or

$$z_k = \begin{bmatrix} \theta \\ \phi \end{bmatrix}_k + \nu_k$$

for the discrete measurement case.

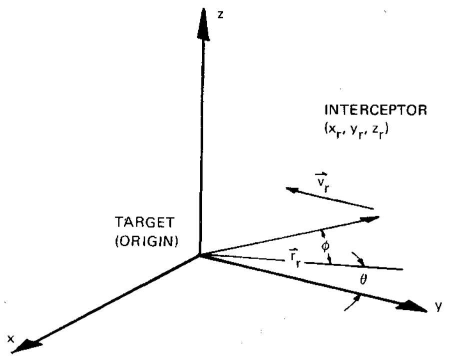

Fig. 1 Typical interceptor-target geometry.

Proportional navigation for homing guidance uses range rate and LOS rate in polar coordinates to calculate directly a guidance acceleration command, a_c, i.e.,

$$a_c \simeq K\dot{r} \times \omega$$

where K is the navigation constant. In the case of the long-range intercept, where poor initialization, angle-measurement degradation, or data dropout may occur, nonlinear filters may be required to provide estimates of the guidance parameters and of the system states. It has been shown,[9] for example, that, when "good" angle measurements are available, nonlinear filters tend to converge better and yield more accurate results when they are implemented in the polar-coordinate frame.

Cartesian-Coordinate Formulation

In the Cartesian-coordinate formulation, the system states include the interceptor target coordinates x, y, z and their derivatives. The dynamics are linear, but the measurements are as follows:

$$\dot{x} = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix} x + \sum_{i=1}^{N} \Delta v_i$$

$$= Ax + \sum_{i=1}^{N} v_i$$

where

$$x^T = [r_x r_y r_z v_x v_y v_z]$$

$$\Delta v_i = [0\ 0\ 0\ \Delta v_x \Delta v_y \Delta v_z]$$

and

$$z = \begin{bmatrix} \theta \\ \phi \end{bmatrix}_{\text{meas}} = \begin{bmatrix} \tan^{-1} \dfrac{-r_x}{r_y} \\ \tan^{-1} \dfrac{r_z}{r_x^2 + r_y^2} \end{bmatrix} + \nu = h(x) + \nu$$

for continuous measurements, and

$$z = \begin{bmatrix} \theta \\ \phi \end{bmatrix} = h_k(x) + \nu_k$$

for discrete measurements.

The simplicity of the transition matrix in this formulation more than compensates for the disadvantages of nonlinear measurements and the traditional guidance approach. This advantage will be used to expedite the covariance prediction

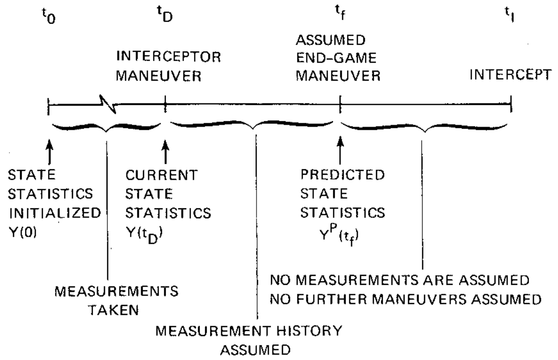

Fig. 2 Two-maneuver guidance law timing diagram.

calculations required to implement the two-maneuver dual control.‡

Predictive Solution to the Homing Problem

A predictive, "two-maneuver" control strategy is proposed here as a suboptimal solution to the homing intercept problem. A timing diagram illustrating the information flow and the allowable maneuvers assumed for the two-maneuver guidance law is shown in Fig. 2.

The purpose of the proposed predictive guidance law is to minimize, or at least reduce, the uncertainty in a given guidance parameter $\eta(x)$ at the assumed terminal maneuver time by making a maneuver (velocity correction or addition) Δv at $t=t_D$. This minimization of future uncertainty is conditioned over all previous knowledge of the state statistics $Y(t_D)$ and over the assumed or predicted statistical information content in the measurements taken over the prediction interval $M(t_f,t_D)$. In addition, it is assumed that the end-game maneuver at $t=t_f$ nulls the residual estimated miss. Specifically, in this formulation, the information function ψ, defined by

$$\psi = E\{[\eta(x)-E(\eta(x))]^2 \mid Y_{t_D}, M(t_f,t_D)\}|_{t_f}$$

or equivalently

$$\psi = E\{[\eta(x)-E(\eta(x))]^2 \mid Y^P(t_f)\}|_{t_f}$$

is minimized with respect to the velocity correction.

This formulation clearly preserves the closed-loop nature inherent in the dynamic-programming solution to the nonlinear stochastic control problem. Calculation of the velocity correction at t_D not only uses information from the past but also uses the fact that more information will be obtained in the future, and that the quality of information obtained by the angle measurements over the prediction interval (t_f, t_D) will be affected by the correction.

Determining the effect of a velocity correction on the predicted variance, or uncertainty, of the chosen guidance parameter η is equivalent to calculating the gradient $(\partial\psi/\partial\Delta v)$. Once the gradient is known, a gradient-search algorithm can be used to find a velocity correction that reduces, or minimizes ψ over an appropriate region of the Δv space. The practical aspects of calculating $(\partial\psi/\partial v)$ are discussed in the following.

Calculation of $(\partial\psi/\partial\Delta v)$

An expression for $(\partial\psi/\partial\Delta v)$ can be obtained in a straightforward manner by making the appropriate linearizations and using Gaussian assumptions. The first assumption is that

$$E[\eta(x)] \simeq \eta(\hat{x}) = \eta[E(x)]$$

‡The polar-coordinate guidance filter still can be used. The information can be transformed easily into Cartesian coordinates for the purposes of these guidance calculations.

A first-order expansion yields

$$\eta(x)-E[\eta(x)] \simeq \eta(x)-\eta(\hat{x})$$

$$\simeq \frac{\partial\eta(x)}{\partial x}\bigg|_{\hat{x}}(x-\hat{x})$$

If we let

$$c^T = \frac{\partial\eta(x)}{\partial x}\bigg|_{\hat{x}}$$

then

$$\eta(x)-E[\eta(x)] = c^T(x-\hat{x})$$

Therefore,

$$\psi \simeq E\{[c^T(x-\hat{x})(x-\hat{x})^T c] \mid Y^P(t_f)\}$$

which is equivalent, by using the definition of the trace operator, to

$$\psi \simeq \operatorname{tr}\{cc^T E[(x-\hat{x})(x-\hat{x})^T \mid Y^P(t_f)]\} \tag{1}$$

By letting the variance $P(t_f)$ be

$$P(t_f) = E[(x-\hat{x})(x-\hat{x})^T \mid Y^P(t_f)]$$

and

$$C(\hat{x}) = cc^T$$

then

$$\psi = \operatorname{tr}\{C(\hat{x})P(t_f)\}$$

Then, defining Δv_i as the ith component of velocity,

$$\frac{\partial\psi}{\partial\Delta v_i} = \operatorname{tr}\left\{\frac{\partial C(\hat{x})}{\partial\Delta v_i}P(t_f) + C(\hat{x})\frac{\partial P(t_f)}{\partial\Delta v_i}\right\} \tag{2}$$

Examination of Eq. (2) shows two terms of equal importance in evaluating the gradient. The first is the product of the variance and the derivative of the sensitivity matrix; the second term is the product of the sensitivity matrix C and the derivative of the variance matrix. It should be clear that a simple reduction in the variance (a decrease in its derivative) may not reduce ψ. In certain cases, a velocity correction that actually increases the error volume relative to other corrections may reduce the guidance-parameter uncertainty, because of the characteristics of the sensitivity-matrix derivative.

The problem of evaluating $P(t_f)$ and $(\partial P/\partial\Delta v)$ (t_f) still remains. These calculations are rather simple when cast in a Cartesian-coordinate framework and when continuous measurements are assumed. First, the measurement equation can be linearized such that

$$\Delta z = h_x \Delta x + \nu$$

where

$$h_x = \frac{\partial h(x)}{\partial x}$$

$\nu(t)$ is assumed to be a zero-mean Gaussian white-noise process with autocorrelation function

$$E[\nu(t)\nu^T(t+\tau)] = R(t)\,\delta(\tau)$$

Over the prediction interval, the mean $\hat{x}$ and variance Σ relations are

$$\hat{x}(t) = \phi(t-t_D)\hat{x}(t_D) = \begin{bmatrix} I & I(t-t_D) \\ 0 & I \end{bmatrix} x(t_D) \tag{3a}$$

$$\dot{\Sigma}=A\Sigma A^T-\Sigma h_x^T R^{-1}(t)h_x\Sigma \tag{3b}$$

Since the actual trajectory over this interval is unknown, h_x is approximated as

$$h_x(x)\simeq h_x[\hat{x}(t)]$$

$P(t)$ will be used to denote the assumed variance over this interval. Substituting P for Σ in Eq. (3), and substituting

$$S=P^{-1}$$

then

$$\dot{S}=-A^TS-SA+h_x^TR^{-1}(t)h_x \tag{4}$$

S can, of course, be solved by the convolution theorem in closed form to yield

$$S(t_f)=\bar{\phi}^*S(t_D)\bar{\phi}^*T+\int_{t_D}^{t_f}\bar{\phi}^*(t_f,\sigma)H(\sigma)\bar{\phi}^{*T}(t_f,\sigma)\mathrm{d}\sigma \tag{5}$$

where

$$H(\sigma)=h_x^T(\sigma)R^{-1}(\sigma)h_x(\sigma)$$

Then,

$$\frac{\partial S}{\partial \Delta v}=\int_{t_D}^{t_f}\bar{\phi}^{*T}(t_f,\sigma)\frac{\partial}{\partial \Delta v}H(\sigma)\bar{\phi}^{*T}(t_f,\sigma)\mathrm{d}\sigma \tag{6}$$

where

$$\bar{\phi}^*=\begin{bmatrix} I & 0 \\ -I(t_f-t_D) & I\end{bmatrix}$$

The integrals in Eqs. (5) and (6) can be evaluated by numerical quadrature without difficulty. It follows immediately that

$$P(t_f)=S^{-1}(t_f)$$

and

$$\frac{\partial P(t_f)}{\partial \Delta v}\simeq -P(t_f)\frac{\partial S(t_f)}{\partial \Delta v}P(t_f)$$

which are the desired quantities.

The motivation for choosing the Cartesian-coordinate system now should be clear. In the Cartesian-coordinate system, the transition matrix is independent of the state, which means that differentiation of Eq. (5) becomes tractable. In fact, only the linearized information rate matrix $H(\sigma)$ needs to be differentiated. Since $h_x(\sigma)$ can be expressed as a function of the state at $t=t_D$ and σ, the differentiation is simple although tedious.§

Note that a priori information regarding the quality of future measurements can be included in $R(\sigma)$. In certain long-range intercept missions, it may be known that measurements may be degraded or even unavailable for parts of the engagement. This causes no computational difficulty and may result only in a higher-order quadrature. It should be emphasized that, if driving noise is present in the form of control errors or random target acceleration, Eq. (4) becomes a nonlinear Ricatti equation and cannot be evaluated easily in closed form.

Guidance Algorithm

It becomes a simple matter to find an incremental velocity correction δv_i that reduces the future guidance-parameter uncertainty. Clearly, by choosing

$$\delta v=-K\frac{\partial \psi}{\partial \Delta v}$$

predicted improvements in performance can be obtained. Successive application of the gradient technique yields a velocity correction that either minimizes ψ or reaches the point of diminishing returns, where $\|(\partial\psi/\partial\Delta v)\|$ becomes small. An important result of this work has been that a point of diminishing returns does exist for moderate velocity corrections. Although a large velocity correction introduces a larger measurable LOS rate, estimation accuracy may not be improved, compared to a smaller correction. In fact, it may degrade performance. This will be discussed in the observations section at the end of this paper.

The discussion thus far has dwelt on a two-maneuver intercept where the first maneuver is applied to enhance the performance of the terminal maneuver. In some engagements, more than two maneuvers may be desired. It is possible to extend the analysis done here to the case of multiple maneuvers, where all but the terminal maneuver are used for estimation enhancement. However, it is also possible to make minor modifications of the two-maneuver dual-control concept so that it can be extended realistically to the multiple-maneuver case.

For multiple burns, it may be assumed that the maneuver times are known. At the time of the ith maneuver t_i, it is assumed that measurements can be taken only up to t_{i+1}. This means that, at t_i, the algorithm expects that the terminal maneuver occurs at t_{i+1}. Restricting the assumed measurement history forces the controller to make estimation accuracy improvements as early in the engagement as possible. Unexpected (unmodeled) track loss or measurement degradation later in the engagement is anticipated effectively, and reacquisition and intercept capabilities are improved.

Application of the Technique

The technique has been applied using three different guidance parameters in the optimization process. The parameters chosen for this study are 1) miss (distance of closest approach), 2) velocity correction, and 3) time-to-intercept (time-to-go). The optimum parameter choice is the one that minimizes the interceptor miss after the terminal correction.¶ No attempt was made to find an optimal information function. However, because nonlinear filter convergence and guidance calculations hinge on the estimates of time-to-intercept, this guidance parameter was considered to be the best choice prior to the numerical analysis, and this was verified.

Assumptions

The following assumptions have been used in running the simulation:

1) An extended Kalman filter implemented in Cartesian coordinates is used to estimate and predict the state.

2) Random target maneuvers or control errors are not considered.

3) The true initial position is

$$r=\begin{bmatrix}0\\1.0\\0\end{bmatrix}$$

§ $\hat{x}(t)$ in the prediction interval is defined as

$$\hat{x}(t)=\hat{x}(t_D)+\hat{v}(t_D)(t-t_D)$$

Using the chain rule, $\partial h_x(t)/\partial v(t_D)$ is defined by

$$\frac{\partial h_x t}{\partial v(t_D)}=\frac{\partial h_x(t)}{\partial \hat{x}(t)}\frac{\mathrm{d}x(t)}{\partial v(t_D)}=(t-t_D)\frac{\partial h_x(t)}{\partial \hat{x}(t)}$$

¶The total miss includes the timing error due to inaccuracies in the time-to-go estimate.

The true initial range is unity.

4) The true initial velocity is

$$v = 0.01 \begin{bmatrix} M_x \\ 0 \\ 0 \end{bmatrix}$$

5) The initial error variance matrix is a diagonal matrix. The positional variance σ_{xx}^2 is $\sigma_{xx}^2 = 10^{-3}$ units.[2] The velocity variance is $\sigma_{\dot{x}\dot{x}}^2 = 10^{-7}$ units^2/s^2.

6) The measurements are made in discrete time at a sampling rate of 1/s. This rate is high enough that little error is introduced in the prediction equations by assuming a continuous measurement process.

7) The measurement variance is assumed uncorrelated, i.e., $R_{\phi\phi} = R_{\psi\psi}$.

8) Three-maneuver engagements are considered. The maneuvers are assumed to be made equidistant in time over the assumed engagement.

9) The gradient-search algorithm is stopped when

$$\left\| \frac{\partial \psi}{\partial \Delta v} \right\| < 0.1 \left\| \frac{\partial \psi}{\partial \Delta v} \right\|_{\max}$$

Generally, the search was stopped in five to six iterations using an adaptive algorithm.

10) The gradient search is initiated after a velocity correction is added to null the estimated miss.

11) A four-point quadrature is used to evaluate the convolution integrals.

12) These results are compared against a guidance law (baseline) that nulls the estimated miss after each maneuver.

Summary of Results

Numerical results obtained using information functions determined by the three different guidance parameters are discussed as follows.

Miss

The information function here is the mean-squared value of the error in estimating the miss (excluding fusing errors). In this formulation, a minimum in ψ occurs extremely close to the baseline trajectory, where the estimated miss is nulled after each maneuver. The small trajectory perturbations introduced by the algorithm are smaller than the uncertainty in making the corrections. Actual miss-estimation accuracy thus is not necessarily improved using the algorithm, because of the discrepancy between the actual change in LOS rate and the estimated change. These results indicate that a discrete proportional navigation law that nulls the miss periodically over an engagement allows for "near-optimal" estimation of the guidance miss.

Velocity Correction

In this case, ψ is chosen to be the mean-squared value of the future error in estimating the velocity correction that nulls the miss. For this control criterion, a minimum of ψ results when a moderate in-plane correction is applied which reverses the initial measurable LOS rate. Table 1 shows typical Monte Carlo results that indicate the relative performance of the dual-control and the baseline guidance laws when the measurement variance R is 10^{-10} rad^2, and when the initial miss M_x is 0.02 unit. The estimated error $\epsilon_{\Delta v}$ is obtained by the relation

$$\epsilon_{\Delta v} = \mathrm{tr}\,[C_{\Delta v} P]$$

where

$C_{\Delta v}$ = sensitivity matrix for the velocity correction
P = error variance matrix

The actual $\epsilon_{\Delta v}$ is the calculated rss error in estimating the velocity correction. As can be seen, the predictive controller introduces a residual miss of roughly one-quarter of the initial miss. By the time of the second maneuver, an estimated improvement of about 25% is observed in estimating the velocity correction. This improvement is less apparent by the time of the terminal maneuver, where the estimated reduction is less than 10%. The actual results are inconclusive in determining which control is better in reducing the actual intercept miss. Total fuel, measured in units of velocity correction, used by this predictive controller is between 60 and 70% higher than the baseline.

As an aside, the dual controller makes improvements in estimating time-to-go (τ). It also should be noted that the chief contributor to the total miss is the error in the time-to-go estimate. For the nominal closing rate v_r of 0.01 unit/s, the timing error $v_r \cdot \Delta\tau$ is at least 1000 times larger than the miss errors due to the correction uncertainty, for all of the cases shown in Table 1.

In summary, it seems that velocity-correction information is available from the measurements regardless of the intercept trajectory. As will be seen, this is not true for range and range-rate information.

Time-to-Go

As stated, the quality of time-to-go information is critical when only angle measurements can be taken. In this formulation, the information function is chosen to be the mean-squared value of the error in estimating time-to-intercept, where

$$|R| / |\dot{R}| = \text{time-to-intercept} = \tau$$

Results for the initial in-plane miss (point of closest approach) $M_x = 0$, 0.02, 0.06, 0.1, using the algorithm, are

Table 1 Typical Monte Carlo results indicating relative performance of dual-control and baseline guidance laws[a]

	$t=25$				$t=50$				$t=75$				
	$\epsilon_{\Delta v}$ ($\times 10^{-6}$)	Actual $\lvert\epsilon_{\Delta v}\rvert$ ($\times 10^{-6}$)	$\lvert\Delta\tau\rvert$	Actual residual miss ($\times 10^{-4}$)	$\epsilon_{\Delta v}$ ($\times 10^{-7}$)	Actual $\lvert\epsilon_{\Delta v}\rvert$ ($\times 10^{-7}$)	$\lvert\Delta\tau\rvert$	Actual residual miss ($\times 10^{-6}$)	$\epsilon_{\Delta v}$ ($\times 10^{-8}$)	Actual $\lvert\epsilon_{\Delta v}\rvert$ ($\times 10^{-8}$)	$\lvert\Delta\tau\rvert$	Terminal miss ($\times 10^{-7}$)	Total fuel ($\times 10^{-4}$)
DC	3.83	10.4	0.59	46.0	1.41	1.27	0.24	108	8.01	5	0.04	9.7	4.40
BL	3.83	10.4	0.59	7.61	1.71	1.27	0.21	1.77	8.48	5.97	0.32	11	3.01
DC	3.98	4	0.93	503	1.73	1.14	0.57	308	9.07	8.05	0.15	24.9	4.22
BL	3.98	4	0.93	3.14	2.51	1.62	1.32	8.74	10.9	7.88	1.13	24.4	2.61
DC	4.23	5.81	0.08	51.6	1.55	0.65	0.59	178	8.52	4.5	0.33	11	4.44
BL	4.23	5.81	0.08	4.3	2.00	0.8	0.32	2.24	9.26	3.44	0.55	8.62	2.81

[a] DC = dual controller with velocity-correction criterion; BL = baseline; three Monte Carlo runs; $R = 10^{-10}$ rad^2; initial miss = 0.02; engagement time ≈ 100 s.

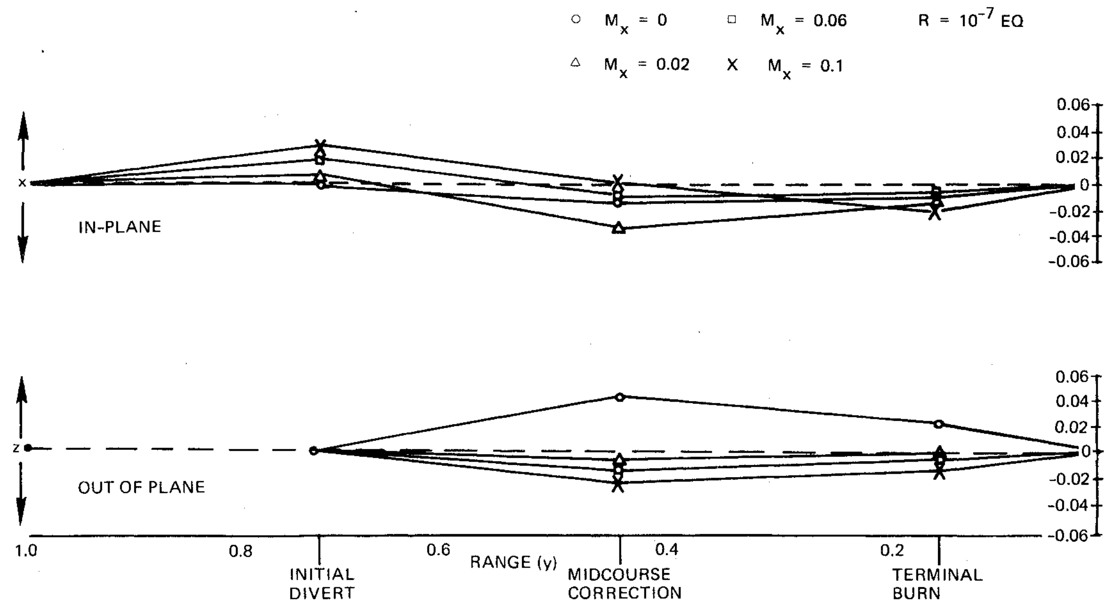

Fig. 3 Dual-controller trajectories with "time-to-go" criterion (sample time history).

shown in Table 2, and the resultant in-plane and out-of-plane trajectories are shown in Fig. 3. The measurement variance is assumed to be 10^{-8} rad. The gradient search is halted for these runs when

$$\left|\left|\frac{\partial\psi}{\partial\Delta v}\right|\right| < 0.1 \left|\left|\frac{\partial\psi}{\partial\Delta v}\right|\right|_{\max}$$

Here the two-maneuver control law, utilizing the time-to-go criterion, makes large multidirectional maneuvers. The dual nature of the control is especially apparent for the cases of small initial miss. In these cases, time-to-go estimation errors prior to the first maneuver are much larger than the 1σ linearized estimates of these errors obtained by the relation

$$\sigma_\tau = \sqrt{\mathrm{tr}(C_\tau P)}$$

where

C_τ = sensitivity matrix for the time-to-intercept function
P = filter variance

This is an indication of filter divergence and results from the minimal initial measurable LOS rate, the measurement quality, and the initial target-designation error. The baseline control scheme continues to null the estimated LOS rate, thereby attenuating the range and range-rate information that can be obtained by the angle measurements. The predictive controller "knows" that range/range-rate information can be obtained by making large maneuvers, and tremendous improvements in the timing estimate are obtained.

Note that a velocity correction in the z direction was introduced in all cases. For all cases except the degenerate case $M_x = 0$, the initial in-plane direction lies in the x-y plane. Statistical variables in the x-y plane are strongly correlated to each other prior to the initial maneuver, and are decoupled almost completely from variables in the orthogonal direction z. The velocity correction and miss formulation of the predictive guidance law indicate that this plane should be maintained by making only in-plane corrective burns. The time-to-go formulation illustrates that more range/range-rate information can be obtained by exercising large out-of-plane

Table 2 Simulated performance of dual controller with time-to-go (τ) criterion[a]

		First maneuver				Second maneuver				Final maneuver				
	Initial miss M_x	τ	$\Delta\tau$	σ_τ	Fuel $\lvert\Delta v\rvert$ $(\times 10^{-5})$	τ	$\Delta\tau$	σ_τ	Fuel $\lvert\Delta v\rvert$ $(\times 10^{-5})$	τ	$\Delta\tau$	σ_τ	Fuel $\lvert\Delta v\rvert$ $(\times 10^{-5})$	Total fuel $\lvert\Delta v\rvert$ $(\times 10^{-5})$
BL	0.0	73.0	−12.7	4.9	0.141	46.2	−13.2	4.9	0.245	19.0	−14.6	5.0	0.142	0.528
DC		73.0	−12.7	4.9	151	42.2	−0.02	0.266	202	14.0	−0.033	0.06	72	426
BL	0.02	72.9	−12.4	4.8	29	46.1	−4.0	3.11	39	18.9	−4.0	2.84	0.14	31
DC		72.9	−12.4	4.8	143	44.4	−0.18	0.34	161	16.5	−0.1	0.07	60	364
BL	0.06	73.0	−3.3	2.75	84.1	45.8	−1.1	1.93	3.08	18.9	−1.4	1.87	0.139	87.4
DC		73.0	−3.3	2.75	177	44.1	−0.02	0.32	142	16.3	−0.003	0.07	53	372
BL	0.1	73.0	−1.2	1.83	177	45.6	−0.23	1.33	3.05	18.6	−0.5	1.28	0.15	142
DC		73.3	−1.2	1.83	220	43.6	−0.20	0.266	140	16.0	−0.02	0.06	56	417

[a] DC = dual controller with time-to-go criterion; BL = baseline; $\Delta\tau = \tau - \hat{\tau}$; $R = 10^{-8}$ rad^2; engagement time = 100 s; sample rate 1/s; initial position variance = 10^{-3}; initial velocity variance = 10^{-7}.

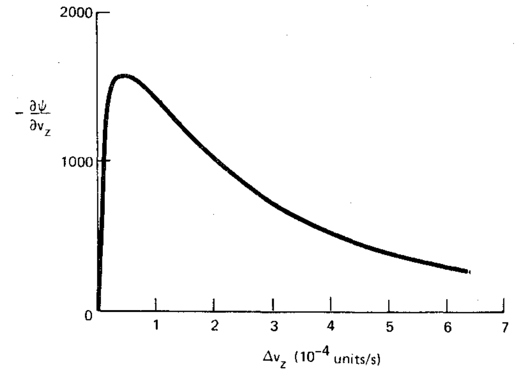

Fig. 4 Sample plot of $(\partial\psi/\partial\Delta v_z)$ for the initial maneuver ($M_x = 0.1$).

maneuvers in conjunction with in-plane maneuvers that reverse the initial LOS rate.

An interesting functional characteristic exposed by this analysis is the fact that the z component of the gradient $(\partial\psi/\partial\Delta v_z)$ around the baseline control is extremely small. After several iterations of the gradient search, this derivative becomes much larger, indicating large-scale performance improvements. A sample plot of $(\partial\psi/\partial\Delta v_z)$ for the initial maneuver with $M_x = 0.1$ is shown in Fig. 4.** This characteristic has far-reaching implications on the capabilities of simple first-order algorithms for making large-scale performance improvements or for effecting suboptimal wide-sense stochastic control laws.

To summarize, substantial performance improvements are obtained using the two-maneuver approach, even in cases of initial filter divergence, where the algorithm performs well enough to reduce terminal timing errors by a factor of 400 compared to the baseline control. In general, the time-to-go error, using the predictive guidance law with $R = 10^{-8}$ rad^2, was at least an order of magnitude better than that obtained by the baseline control with $R = 10^{-10}$ rad^2. It is true that larger amounts of fuel are used. It should be clear, however, that rough fuel constraints could be factored into the problem to stop the gradient search when the fuel required to make a maneuver and to correct for it later in the engagement exceeds the constraint.

Some Observations

Exhaustive Monte Carlo testing of the technique was not done. Nonetheless, several observations and conclusions regarding the information flow into angle-only processors, as well as the control technique itself, can be made.

The control technique, which separates the function of the maneuvers in a two-maneuver intercept into estimation enhancement and terminal control, naturally captures the key properties of the dual-control solution of the intercept problem. The formulation enhances the estimates of the user-selected guidance parameter that is most crucial for optimum performance of the user-designed terminal intercept scheme. The technique can be adapted easily to the more practical nonimpulse discrete maneuver case. This can be done by introducing a missile acceleration component of the form

$$a = -K\frac{\partial\psi}{\partial\Delta v}$$

or by specifying a guidance miss (obtained by the gradient algorithm) to the interceptor autopilot. Off-line computations and rules of thumb developed through simulation also can be used to simplify computational complexity if needed. No attempt has been made to do this here.

**Note that the curve is symmetric with respect to Δv_z.

There exists a system tradeoff between fuel consumption and required angle-measurement accuracy. The results in this paper have shown that, by using relatively poor angle measurements in conjunction with the dual-control guidance law, the ultimate intercept performance may be superior to a traditional homing system using fine measurements. Much of the work being performed in the technical community to reduce the tracking measurement errors may be unnecessary for homing interceptors if the proper guidance concept is used to enhance the information content of the measurements.

The characteristics of the optimal intercept trajectory are a strong function of the type of information which is to be obtained by making the maneuvers. In addition, enhancement of one parameter estimate usually results in poorer estimates of other parameters. For example, enhancing miss estimates degrades velocity correction information, and vice versa.

It generally is conceded that introducing or leaving in a residual miss after early maneuvers is desirable. This work has shown that the amount and direction of the residual miss is important in determining the type of information which can be extracted from the angle measurements. In fact, one of the important results of this work has been the discovery that inappropriate introduction of large LOS rates actually may degrade estimates of the important guidance parameters. More is not necessarily better. Arbitrary artificial limiting of the controls by quadratic penalty functions (as in Refs. 2-5) may be unnecessary, since moderate velocity corrections have been shown to directly minimize (reach the point of diminishing returns) several chosen information functions important to terminal intercept performance.

One-step first-order optimal-control algorithms probably will not perform comparably to wide-sense algorithms like this one. The components of the gradient of the information function may be small around nominal zero-miss trajectories, even when regions of steep descent or a minimum are nearby. Velocity corrections that, in a strong sense, enhance the incoming angle information content may not be found by a first-order search.

Conclusions

The proposed two-maneuver dual-control guidance technique directly incorporates navigational uncertainties into the guidance formulation. The first maneuver is used to enhance a user-specified guidance parameter estimate, while the second is used to null the intercept miss. It has been shown that the velocity correction in the first maneuver can be obtained by a gradient search that minimizes or substantially reduces the expected variance in the chosen guidance parameter. It also has been shown that the technique can be extended to multiple-maneuver engagements and has the dual-control properties inherent in optimal stochastic control solutions of the intercept problem. The technique can be used as an analysis tool to study the information flow in angle-only processors for intercept missions. More important, the technique can be applied to a multitude of intercept scenarios and is compatible with traditional guidance approaches. Most important, the guidance law does not require extreme angle-measurement accuracy required by the traditional guidance approaches to maintain intercept performance.

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